hep-th/9702116
DAMTP/97-9

Non-confinement in Three Dimensional Supersymmetric Yang–Mills Theory

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Abstract

The role of instantons in three dimensional N=2 supersymmetric $SU(2)$ Yang–Mills theory is studied, especially in relation to the issue of confinement. The instanton-induced low energy effective action is derived by extending the dilute gas approximation to the super-moduli space of instantons. Following Polyakov's description of confinement in compact $U(1)$ gauge theory, it is argued that there is no confinement in N=2 supersymmetric Yang–Mills theory.

1 Introduction

Confinement in three dimensional compact $U(1)$ gauge theory was demonstrated analytically by Polyakov [1] in 1977. He showed that the Wilson loop has an area dependence which arises from instanton effects, where the instanton is the 't Hooft–Polyakov monopole. The photon becomes massive by instanton condensation so a mass gap is generated.

The main purpose of this work is to extend the analysis to a supersymmetric $SU(2)$ Yang–Mills theory without matter multiplets in which the instanton is a BPS monopole. Since monopole solutions require Higgs fields, we need, at least, N=2 supersymmetry. BPS monopoles preserve half the supersymmetry and the broken supersymmetry generates fermionic zero modes. This is also true when the monopoles are interpreted as instantons in three dimensions. These configurations also have very particular dynamics compared to generic 't Hooft–Polyakov monopoles [2]. In the BPS limit of the bosonic theory, this leads to a somewhat singular limit of Polyakov's considerations. We will see that a close relation emerges between the instanton-induced low energy effective action and a complex Toda action [3] in three dimensions.

In the N=2 supersymmetric theory, the low energy effective action can be written in terms of a photon, a Higgs scalar and a complex fermion field [4]. This action was obtained in [4] from symmetry arguments and calculating instanton contributions to a fermion propagator. But it can be explicitly derived by extending the dilute gas approximation to the superspace consisting of collective coordinates of BPS monopoles. Such a dilute gas approximation is more appropriate for a monopole gas than the BPST instanton gas in four dimensions since the size of monopoles is fixed by the parameters of the theory. It can be observed from the effective action that there is no mass gap in the supersymmetric theory, which could be a sign of non-confinement. Indeed, we will see by the Wilson loop criterion that there is no confinement in N=2 supersymmetric Yang–Mills theory.

The low energy effective action of the bosonic theory in the BPS limit is given in section 2 and the question of confinement is discussed. The supersymmetric case is discussed in section 3. Some features of the BPS monopole as an instanton solution of the three dimensional system are reviewed in Appendix.

2 The BPS limit of the bosonic theory

Before embarking on the supersymmetric theory, we will discuss the BPS limit of Polyakov's arguments in [1], where he studied $SU(2)$ Yang–Mills–Higgs theory spontaneously broken to $U(1)$. The partition function of interest is given by, in $\mathbf{R}^3$,

$$\mathcal{Z} = \int \mathcal{D}A_i \mathcal{D}\Phi \exp\Big[-\int d^3x \; Tr\Big(\frac{1}{4}F_{ij}F^{ij} + \frac{1}{2}D_i^2\Phi\Big)\Big] \tag{1}$$

where Φ is the Higgs field in the adjoint representation of $SU(2)$ and $i, j = 1, 2, 3$.

The action for a multi-instanton configuration is the energy of the corresponding static multi-BPS monopole configuration in four dimensions, which gives, in a dilute gas approximation,

$$\mathcal{Z} = \sum_{N,q(=\pm 1)} \frac{\zeta^N}{N!} \int \prod_{i=1}^{N} d^3\mathrm{R}_i \exp\Big(-\frac{\pi}{2e^2} \sum_{i\neq j}^{N} \frac{q_i q_j - 1}{|\mathrm{R}_i - \mathrm{R}_j|} \Big) \tag{2}$$

where $\zeta = c\, e^{-4\pi v/e}$, v is the vacuum expectation value of Φ and c is the one-loop functional determinant in a single BPS monopole background. The arguments in the exponent account for the fact that there is no force between like charges and the attraction between opposite charges is double the Coulomb force. These multi-BPS monopole configurations satisfy the Bogomol'nyi equation only when they consist of like charges. But the mixed combination of BPS monopoles and anti-BPS monopoles are still good approximate solutions of the equations of motion.

The partition function can be written, using Gaussian integral identities, as a functional integral over two real scalar fields, representing the photon and the Higgs scalar,

$$\begin{aligned}
\mathcal{Z} &= C \sum_{N,q(=\pm 1)} \frac{\zeta^N}{N!} \int \prod_{i=1}^{N} d^3\mathrm{R}_i \int \mathcal{D}\gamma\mathcal{D}\phi \; e^{-(\frac{e}{2\pi})^2 \int d^3x \{\frac{1}{2}(\nabla\gamma)^2 + \frac{1}{2}(\nabla\phi)^2\} + i\sum_{i=1}^{N} q_i\gamma(\mathrm{R}_i) - \sum_{i=1}^{N}\phi(\mathrm{R}_i)} \\
&= C \int \mathcal{D}\gamma\mathcal{D}\phi \; e^{-(\frac{e}{2\pi})^2 \int d^3x\{\frac{1}{2}(\nabla\gamma)^2 + \frac{1}{2}(\nabla\phi)^2\}} \sum_{N} \frac{\zeta^N}{N!} \int \prod_{i=1}^{N} \Big(d^3\mathrm{R}_i \; 2\cos\gamma(\mathrm{R}_i) e^{-\phi(\mathrm{R}_i)} \Big) \\
&= C \int \mathcal{D}\gamma\mathcal{D}\phi \; \exp\Big[-(\frac{e}{2\pi})^2 \int d^3x \Big\{ \frac{1}{2}(\nabla\gamma)^2 + \frac{1}{2}(\nabla\phi)^2 - \frac{m^2}{2}(e^{-\phi+i\gamma} + e^{-\phi-i\gamma}) \Big\} \Big]
\end{aligned} \tag{3}$$

where $C = \det| -\frac{e^2}{2\pi^2}\nabla^2|$ and $m^2 = 2\zeta(\frac{2\pi}{e})^2$. The instanton-induced effective action in (3) is a complex Toda-like action [3] in three dimensions. It has a rather singular behaviour caused by the $e^{-\phi}$ term which makes the system unstable. But the effective action will be useful for comparison with that of the supersymmetric theory to be considered in the next section.

The Wilson loop can be calculated, following [1], and is given in the semi-classical approximation as

$$\begin{aligned}
W &\equiv \Big\langle \exp\Big(i \oint_{\Gamma} A_i^3 dx^i \Big) \Big\rangle = \Big\langle \exp\Big(i \int d^3x \, \rho(\mathbf{x})\eta(\mathbf{x}) \Big) \Big\rangle \\
&= \exp\Big[-(\frac{e}{2\pi})^2 \int d^3x \Big\{ \frac{1}{2}(\nabla\gamma_c - \nabla\eta)^2 + \frac{1}{2}(\nabla\phi_c)^2 - m^2 \cos\gamma_c \, e^{-\phi_c} \Big\} \Big]
\end{aligned} \tag{4}$$

where the loop Γ is located in the x_1-x_2 plane, ρ is the monopole charge density and η is defined by

$$\eta(\mathbf{x}) = \frac{1}{2} \int_S d\mathbf{S}_y^i \frac{(\mathbf{y}-\mathbf{x})^i}{|\mathbf{y}-\mathbf{x}|^3} \quad . \tag{5}$$

The fields γ_c, ϕ_c satisfy

$$\nabla^2\gamma_c = \nabla^2\eta + m^2 \sin\gamma_c \, e^{-\phi_c}$$

$$
\begin{aligned}
&= -2\pi\delta'(x_3)\theta_S(x_1,x_2) + m^2 \sin\gamma_c\, e^{-\phi_c} \\
\nabla^2\phi_c &= m^2 \cos\gamma_c\, e^{-\phi_c}
\end{aligned} \tag{6}
$$

where S is the minimal surface whose boundary is the loop Γ and

$$
\begin{aligned}
\theta_S(x_1,x_2) &= 1 \qquad \text{if} \quad (x_1,x_2) \in S\,, \\
&= 0 \qquad \text{otherwise}\,.
\end{aligned} \tag{7}
$$

For a large loop with order of R^2, the system is essentially one-dimensional for $x_1^2+x_2^2 \ll R^2$ and (6) can be reduced to

$$
\frac{d^2\gamma_c}{dx_3^2} = m^2 \sin\gamma_c\, e^{-\phi_c}, \qquad \frac{d^2\phi_c}{dx_3^2} = m^2 \cos\gamma_c\, e^{-\phi_c}\,. \tag{8}
$$

Eq.(8) should be solved to check confinement but an analytic solution has not been found. For our purposes, this is only part of the complete supersymmetric theory to which we now turn.

3 N=2 Supersymmetric Yang–Mills theory

Inclusion of dynamical fermions can cause problems for the the Wilson loop criterion of confinement [5]. But the dynamical fermions in supersymmetric Yang–Mills theories are in the adjoint representation of the gauge group, whereas the test charges are in the fundamental representation.

The partition function of N=2 supersymmetric Yang–Mills theory in Euclidean space is

$$
\mathcal{Z} = \int \mathcal{D}A_i \mathcal{D}\Phi\mathcal{D}\Psi^*\mathcal{D}\Psi \exp\Big[-\int d^3x\; Tr\Big(\frac{1}{4}F_{ij}F^{ij} + \frac{1}{2}D_i^2\Phi + \Psi^\dagger(i\not{D} + e[\Phi,\,])\,\Psi \Big)\Big]. \tag{9}
$$

where Ψ, Ψ^* must be treated as independent Dirac spinors. The supersymmetry transformations are

$$
\begin{aligned}
\delta A_i &= i\Psi^\dagger\sigma_i\alpha - i\alpha^\dagger\sigma_i\Psi \\
\delta\Phi &= i\Psi^\dagger\alpha - i\alpha^\dagger\Psi \\
\delta\Psi &= i(\not{B} - \not{D}\Phi)\alpha, \qquad \delta\Psi^* = -i(\not{B} + \not{D}\Phi)\alpha^*\,.
\end{aligned} \tag{10}
$$

In addition to the translational zero modes, instantons now have fermionic zero modes generated by the supersymmetry transformations (10). These fermionic zero modes need to be included in the effective action calculation. The superfield formalism turns out to be efficient for this purpose [6]. Collective coordinates of anti-instantons and instantons will be denoted as (x_i, α_i) and $(\mathrm{y}_i, \alpha_i^*)$, respectively. The instanton superfield V_S with a collective coordinate (y, α^*) can be written as

$$
V_S(\mathrm{y}, \alpha^*) = e^{-i\mathrm{y}\cdot P + i\alpha^* Q^*} V_B(\mathrm{y}=0) \tag{11}
$$

where V_B represents the bosonic instanton configuration. Under the complex supersymmetry transformation with an algebra $\{Q, Q^*\} = 4\,\sigma\cdot P$,

$$
\begin{aligned}
V_S &\rightarrow e^{i\epsilon Q + i\epsilon^* Q^*} e^{-i\mathrm{y}\cdot P + i\alpha^* Q^*} V_B \\
&= \exp\{-i\,(\mathrm{y} - 2i\epsilon\sigma\alpha^*)\cdot P + i\,(\alpha^* + \epsilon^*)Q^*\} V_B
\end{aligned}
\tag{12}
$$

where $\sigma = (\sigma_1, \sigma_2, \sigma_3)$. Hence, supersymmetry transformations induce translations of instanton collective coordinates as

$$
\delta\mathrm{y}_j^a = -2i\epsilon\sigma^a\alpha_j^*, \qquad \delta\alpha_j^* = \epsilon^*\ . \qquad (a = 1, 2, 3)
\tag{13}
$$

Those for anti-instantons can be obtained in the same way, which are

$$
\delta\mathrm{x}_i^a = 2i\alpha_i\sigma^a\epsilon^*, \qquad \delta\alpha_i = \epsilon\ .
\tag{14}
$$

Now the instanton contribution can be represented by the partition function of the dilute instanton gas in the superspace consisting of their collective coordinates [7]. The remaining task is to supersymmetrize the Coulomb potential of the bosonic theory. The supersymmetry-invariant distance between (x_i, α_i) and $(\mathrm{y}_i, \alpha_i^*)$ is

$$
|\mathrm{x}_i - \mathrm{y}_j - 2i\alpha_i\sigma\alpha_j^*|
\tag{15}
$$

which is easy to check using (13) and (14). The partition function (9) can, then, be written as

$$
\begin{aligned}
\mathcal{Z} &= \sum_{M,N} \frac{\zeta^{M+N}}{M!N!} \int \prod_{i=1}^{M} d^3\mathrm{x}_i d^2\alpha_i \prod_{j=1}^{N} d^3\mathrm{y}_j d^2\alpha_j^* \exp\Big(\frac{\pi}{e^2} \sum_{i,j}^{M,N} \frac{1}{|\mathrm{x}_i - \mathrm{y}_j - 2i\alpha_i\sigma\alpha_j^*|}\Big) \\
&= \sum_{M,N} \frac{\zeta^{M+N}}{M!N!} \int \prod_{i=1}^{M} d^3\mathrm{x}_i d^2\alpha_i \prod_{j=1}^{N} d^3\mathrm{y}_j d^2\alpha_j^* \exp\Big(\frac{\pi}{e^2} \sum_{i,j}^{M,N} \frac{1}{|\mathrm{x}_i - \mathrm{y}_j|}\Big(1 + 2i\,\Big|\frac{\alpha_i\sigma\alpha_j^*}{\mathrm{x}_i - \mathrm{y}_j}\Big|\Big) \\
&= \sum_{M,N} \frac{\zeta^{M+N}}{M!N!} \int \prod_{i=1}^{M} d^3\mathrm{x}_i d^2\alpha_i \prod_{j=1}^{N} d^3\mathrm{y}_j d^2\alpha_j^* \exp\Big(\frac{\pi}{e^2} \sum_{i,j}^{M,N} e^{-2i\alpha_i\sigma\alpha_j^* \frac{\partial}{\partial \mathrm{x}_i}} \frac{1}{|\mathrm{x}_i - \mathrm{y}_j|}\Big)
\end{aligned}
\tag{16}
$$

where the potential energy comes only from the oppositely charged BPS monopoles.

As in the bosonic theory, the exponent can be expressed as a functional integral, this time, over a N=2 scalar superfield,

$$
\begin{aligned}
\mathcal{Z} = &\sum_{M,N} \frac{\zeta^{M+N}}{M!N!} \int \prod_{i=1}^{M} d^3\mathrm{x}_i d^2\alpha_i \prod_{j=1}^{N} d^3\mathrm{y}_j d^2\alpha_j^* \int \mathcal{D}\Phi^*\mathcal{D}\Phi\, \exp\Big[\int d^3x d^2\theta d^2\theta^*\, \Phi^*\Phi \\
&- \sum_{i=1}^{M} \frac{2\pi}{e}\, \Phi(\mathrm{x}_i, \alpha_i) - \sum_{j=1}^{N} \frac{2\pi}{e}\, \Phi^*(\mathrm{y}_j, \alpha_j^*)\,\Big] \\
= &\int \mathcal{D}\Phi^*\mathcal{D}\Phi\, \exp\Big[\int d^3x \Big\{ \int d^2\theta d^2\theta^*\, \Phi^*\Phi + \int d^2\theta\, \mathcal{W}(\Phi) + \int d^2\theta^*\, \mathcal{W}(\Phi^*) \Big\}\Big]
\end{aligned}
\tag{17}
$$

where the non-perturbative superpotential

$$\mathcal{W}(\mathit{\Phi}) = \zeta \, \exp\Big(- \frac{2\pi}{e}\, \mathit{\Phi} \Big). \tag{18}$$

As can be expected from the close relation between the effective action of the bosonic theory and the complex Toda action, the action in (17) is that of the N=2 supersymmetric A_1 Toda theory in three dimensions [8].

Defining $\mathit{\Phi} \equiv Z + \sqrt{2}\theta\psi + \theta\theta F$ and integrating out the auxiliary field F, the instanton-induced effective lagrangian can be recast in terms of component fields as

$$\mathcal{L} = \partial_i Z^* \partial^i Z + i\psi^\dagger \partial\!\!\!/ \,\psi + \frac{1}{2}\zeta\Big(\frac{2\pi}{e}\Big)^2 \Big(\psi\psi e^{-\frac{2\pi}{e}Z} + \psi^*\psi^* e^{-\frac{2\pi}{e}Z^*}\Big) + \Big(\frac{2\pi}{e}\zeta\Big)^2 e^{-\frac{2\pi}{e}(Z+Z^*)} \tag{19}$$

where $\mathcal{L}$ is defined by

$$\mathcal{Z} = \int \mathcal{D}^2 Z \mathcal{D}\psi^* \mathcal{D}\psi \mathcal{D}^2 F \, \exp\Big(- \int d^3x \, \mathcal{L} \Big) \, . \tag{20}$$

Thus, the dilute gas approximation in superspace has enabled us to derive the bosonic potential term in (19) which was motivated in [4] by requiring the effective action to be supersymmetric. The potential terms in (3) are now coupled with the fermionic terms because of the fermionic zero-modes of BPS monopoles.

Following (4), the Wilson loop is

$$\begin{aligned} W &= \Big\langle e^{i\int d^3x\, \rho(x)\eta(x)} \Big\rangle \\ &= \sum_{M,N} \frac{\zeta^{M+N}}{M!N!} \int \prod_{i=1}^{M} d^3\mathrm{x}_i d^2\alpha_i \prod_{j=1}^{N} d^3\mathrm{y}_j d^2\alpha_j^* \int \mathcal{D}\mathit{\Phi}^* \mathcal{D}\mathit{\Phi} \, \exp\Big[\int d^3x d^2\theta d^2\theta^* \, \mathit{\Phi}^*\mathit{\Phi} \\ &\quad - \sum_{i=1}^{M} \Big\{ \frac{2\pi}{e} \mathit{\Phi}(\mathrm{x}_i, \alpha_i) + i\eta(\mathrm{x}_i) \Big\} - \sum_{j=1}^{N} \Big\{ \frac{2\pi}{e} \mathit{\Phi}^*(\mathrm{y}_j, \alpha_j^*) - i\eta(\mathrm{y}_j) \Big\} \Big] \\ &= \int \mathcal{D}^2\mathit{\Phi} \, \exp\Big[\int d^3x \Big\{ \int d^2\theta d^2\theta^* \, \mathit{\Phi}^*\mathit{\Phi} + \int d^2\theta \, e^{-\frac{2\pi}{e}\mathit{\Phi} - i\eta} + \int d^2\theta^* \, e^{-\frac{2\pi}{e}\mathit{\Phi}^* + i\eta} \Big\} \Big] \end{aligned} \tag{21}$$

In terms of component fields, (21) can be expressed as

$$\begin{aligned} W &= \int \mathcal{D}^2 Z \mathcal{D}\psi^* \mathcal{D}\psi \, \exp\Big[- \int d^3x \, \Big\{ \Big(\nabla Z - i\frac{e}{2\pi}\nabla\eta \Big)^2 \\ &\quad + \frac{1}{2}\zeta\Big(\frac{2\pi}{e}\Big)^2 \Big(\psi\psi e^{-\frac{2\pi}{e}Z} + \psi^*\psi^* e^{-\frac{2\pi}{e}Z^*}\Big) + \Big(\frac{2\pi}{e}\zeta\Big)^2 e^{-\frac{2\pi}{e}(Z+Z^*)} \Big\} \Big] \, . \end{aligned} \tag{22}$$

Integrating out ψ, ψ^* gives an effective potential $f\Big(e^{-\frac{2\pi}{e}(Z+Z^*)}\Big)$ whose explicit form will be unimportant for the discussion. It is important that f depends only on the real part of the complex scalar field Z. Therefore, we can write (22) as

$$W = \int \mathcal{D}\gamma \mathcal{D}\phi \, \exp\Big[- \int d^3x \, \Big\{ \frac{1}{2}\Big(\nabla\gamma - \frac{e}{\sqrt{2}\pi}\nabla\eta \Big)^2 + + \frac{1}{2}(\nabla\phi)^2 + U(\phi) \Big\} \Big] \tag{23}$$

where ϕ and γ are defined by $Z = \frac{1}{\sqrt{2}}(\phi + i\gamma)$ and $U(\phi) = \left(\frac{2\pi}{e}\zeta\right)^2 e^{-\frac{2\sqrt{2}\pi}{e}\phi} + f(\phi)$. In the semi-classical approximation, (22) is given by

$$W = \exp\Big[- \int d^3x \Big\{ \frac{1}{2}\Big(\nabla\gamma_c - \frac{e}{\sqrt{2}\pi}\nabla\eta\Big)^2 + \frac{1}{2}(\nabla\phi_c)^2 + U(\phi_c)\Big\}\Big] \tag{24}$$

where γ_c and ϕ_c satisfy

$$\begin{aligned} \nabla^2\gamma_c &= \frac{e}{\sqrt{2}\pi}\nabla^2\eta \\ \nabla^2\phi_c &= \frac{dU}{d\phi_c}. \end{aligned} \tag{25}$$

Now it can be seen from (25) that ϕ_c is completely decoupled from the source term. It will only give some overall numerical factor. The classical field γ_c is analogous to the electric potential due to a dipole layer in electrodynamics [9]. By partial integration, (24) can be written as

$$W = N \exp\Big[\frac{1}{2}\int d^3x \Big(\gamma_c - \frac{e}{\sqrt{2}\pi}\eta\Big)\Big(\nabla^2\gamma_c - \frac{e}{\sqrt{2}\pi}\nabla^2\eta\Big)\Big] \tag{26}$$

where N represents various numerical factors. From (25), the argument in the exponent vanishes and there is no area law behaviour. The fluctuation around the classical configuration was argued in [9] to give the Wilson loop a perimeter dependence. Hence, there is no confinement in N=2 supersymmetric Yang–Mills theory. This could also be anticipated from the effective action (19), where instanton effects make the fermions massive but the photon field γ remains massless.

In summary, we have derived the instanton-induced effective action of the N=2 supersymmetric Yang–Mills theory in a manner that parallels the discussion of the non-supersymmetric theory in [1]. In the presence of the extended supersymmetry, the instanton gas does not lead to confinement. The essential feature is that the non-perturbative potential which makes a photon massive in the bosonic theory does not occur in the supersymmetric case. Instanton effects generate mass terms only for fermions. In [10], the N=4 supersymmetric Yang–Mills theory without matter multiplets was also argued to have no confinement.

Acknowledgements: I would like to thank Michael Green for suggesting the problem, insightful discussions and encouragements. I also appreciate discussions with Jonathan Evans, Michael Gutperle and Nick Manton on some issues in the paper. This work was supported by COT, ORS Award and Dong-yung Scholarship.

Appendix : BPS monopoles as instantons in three dimensions

't Hooft–Polyakov monopoles arise as instanton solutions of the three dimensional $SU(2)$ Yang–Mills–Higgs theory spontaneously broken to $U(1)$ whose action in $\mathbf{R}^3$ is

$$\mathcal{S} = \int d^3x \; Tr\Big(\frac{1}{4}F_{ij}F^{ij} + \frac{1}{2}D_i^2\Phi + \frac{\lambda}{4}(\Phi^2 - a^2)^2 \Big) \; . \tag{27}$$

It can be easily seen that, in the BPS limit ($\lambda = 0$), the action is minimised by the field configuration satisfying

$$B_i = \pm D_i\Phi \, , \tag{28}$$

where we defined $B_i = \frac{1}{2}\epsilon_{ijk}F^{jk}$, the dual of F_{ij}. Eq.(28) is known as the Bogomol'nyi equation whose solution is the BPS monopole.

Although these instantons are simply BPS monopole configurations in four dimensions, they have certain features particular to three dimensions. Since there is no extra time coordinate on which the fields may depend, there is no analogue of the dyon solutions of four dimensions which depend on time. In other words, there is no A_0 field that characterises the dyon solutions [11]. We can ask, however, whether they have an electric charge in Euclidean sense, an unbroken $U(1)$ charge. This $U(1)$ gauge transformation is the rotation around $\Phi/|\Phi|$ [12]. Under the infinitesimal gauge transformation, the fields transform as

$$\delta\Phi = 0, \qquad \delta A_i = -\frac{1}{ev}D_i\Phi \tag{29}$$

where v is the vacuum expectation value of Φ. In Euclidean space, the Noether current is

$$J^i = \frac{1}{ev}Tr\Big(\epsilon_{ijk}B^jD^k\Phi \Big) \tag{30}$$

hence, by the Bogomol'nyi equation (28), the $U(1)$ charge vanishes for BPS monopoles. But non-BPS monopoles can have non-zero $U(1)$ charge. There is no contribution from a θ-term to the $U(1)$ current since the θ-term of the theory is

$$\mathcal{L}_\theta = i\frac{e\theta}{4\pi v}Tr\Big(B^iD_i\Phi \Big) \tag{31}$$

and its contribution to the current J^i is

$$\begin{aligned} J^i_\theta &= i\frac{\theta}{4\pi v^2}\epsilon^{ijk} \; Tr\Big(D_j\Phi D_k\Phi \Big) \\ &= 0 \; . \end{aligned} \tag{32}$$

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